Viba Classes

RELATION AND FUNCTIONS

a) Empty Relation

$R = \phi$

b) Universal Relation

$R = A \times B$

1) Reflexive Relation

If $(a,a) \in R$

2) Symmetric Relation

If $(a,b) \in R$ then $(b,a) \in R$

3) Transitive Relation

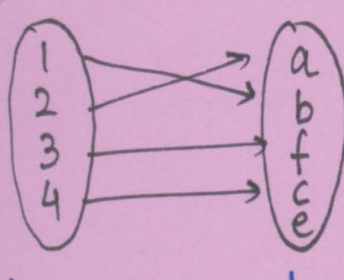
If $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

4) If Relation

Reflexive + Symmetric + Transitive = Equivalence Relation

5) One-one function / Injective

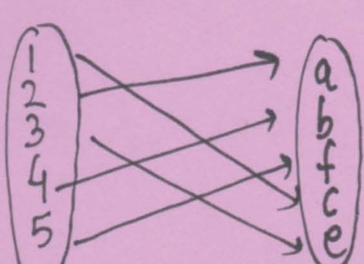
If every single Range has unique Domain function, it is said to be one-one function



one-one but not onto function

6) onto function / Surjective

If Codomain = Range then function is termed as Onto function



one-one as well as onto function

one-one + onto Bijective

VIBA CLASSES

FORMULAS OF TRIGONOMETRY

10th

1.) $\sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\theta = \cos^2\theta$

$1 - \cos^2\theta = \sin^2\theta$

2.) $1 + \tan^2\theta = \sec^2\theta$

$\sec^2\theta - 1 = \tan^2\theta$

$\sec^2\theta - \tan^2\theta = 1$

3.) $1 + \cot^2\theta = \operatorname{cosec}^2\theta$

$\operatorname{cosec}^2\theta - 1 = \cot^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

4.) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

5.) $\sec\theta = \dfrac{1}{\cos\theta}$, $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$, $\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

11th

Category 1.

Degree measure = Radian measure $\times \dfrac{180^\circ}{\pi}$

Radian measure = Degree measure $\times \dfrac{\pi}{180^\circ}$

$\theta = \dfrac{l}{r}$

Category 2.

$\sin(A+B) + \sin(A-B) = 2\sin A\cos B$

$\sin(A+B) - \sin(A-B) = 2\cos A\sin B$

$\cos(A+B) + \cos(A-B) = 2\cos A\cos B$

$\cos(A+B) - \cos(A-B) = -2\sin A\sin B$

$A > B$

Category 3.

$\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$

$\cot(A+B) = \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$, $\cot(A-B) = \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$

Category 4

$$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

$$\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$$

$$\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$$

Category 5 Half Angle Formulas

1. $\sin\theta = 2\sin\theta/2 \cos\theta/2 = \dfrac{2\tan\theta/2}{1+\tan^2\theta/2}$

2. $\cos\theta = \cos^2\dfrac{\theta}{2} - \sin^2\dfrac{\theta}{2} = 2\cos^2\dfrac{\theta}{2} - 1 = 1 - 2\sin^2\dfrac{\theta}{2}$

3. $\tan\theta = \dfrac{2\tan\theta/2}{1-\tan^2\theta/2}$

Category 6 Golden Formula

1. $1+\cos\theta = 2\cos^2\theta/2$

   $1-\cos\theta = 2\sin^2\dfrac{\theta}{2}$

Category 7 1/3rd of Angle Formulas

1. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

2. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

3. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

Category 8 General solution

1. If $\sin\theta = 0$ then $\theta = n\pi$, $n \in Z$
2. If $\cos\theta = 0$ then $\theta = (2n+1)\frac{\pi}{2}$, $n \in Z$
3. If $\tan\theta = 0$ then $\theta = n\pi$, $n \in Z$
4. If $\sin\theta = \sin\alpha$ then $\theta = n\pi + (-1)^n \alpha$, $n \in Z$
5. If $\cos\theta = \cos\alpha$ then $\theta = 2n\pi \pm \alpha$, $n \in Z$
6. If $\tan\theta = \tan\alpha$ $\theta = n\pi + \alpha$, $n \in Z$

Class 12th

1. $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

   $\sec^{-1}x + \operatorname{cosec}^{-1}x = \frac{\pi}{2}$

   $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

2.) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

   $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

3. $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

4. $\sin^{-1}(-x) = -\sin^{-1}x$

   $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1}x$

   $\tan^{-1}(-x) = -\tan^{-1}x$

   $\cos^{-1}(-x) = \pi - \cos^{-1}x$

   $\sec^{-1}(-x) = \pi - \sec^{-1}x$

   $\cot^{-1}(-x) = \pi - \cot^{-1}x$

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9) Zero Matrix – A matrix is said to be zero matrix when all its elements are zeroes.
For example – $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Two matrices are said to be equal when they have same order and elements.

Matrices obey the following properties during ADDITION –

1. Commutative Law = $A + B = B + A$
2. Associative Law : $(A+B) + C = A + (B+C)$
3. Existence of Additive Identity = $A + 0 = A$
4. Existence of Additive Inverse = $A + (-A) = 0$

Matrices obey the following properties during MULTIPLICATION –

1. Associative Law = $(AB)C = A(BC)$
2. Distributive Law = $A(B+C) = AB + AC$
3. The Existence of Multiplicative Identity = $AI = A$

Matrix division by a scalar is possible but not by another matrix.

When the rows and columns of a given matrix are interchangeable, then the resultant matrix is called TRANSPOSE of the given matrix.

Matrices obey the following properties during TRANSPOSE:

1.) $(A')' = A$
2.) $(A+B)' = A' + B'$
3.) $(KA)' = KA'$ (K = constant)
4.) $(AB)' = B'A'$
5.) $(A-B)' = A' - B'$

The Inverse of given matrix if its exists; is

MATRICES

There are Seven types of Matrices given as follows -

a) Column Matrix - A matrix is said to be column matrix if it has only one column.

For example = $A = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$ is a column matrix of order 3X1

b) Row Matrix - A matrix is defined as Row matrix when it consists of only one Row.

For example = $A = [1, 5\ 8]$ is a row matrix of order 1X3

c) Square Matrix - A matrix in which number of rows are equal to number of columns.

For example = $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$ is a square matrix.

d) Diagonal Matrix - A square matrix is said to be diagonal matrix when all its non-diagonal elements are zeroes.

For example = $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ is a diagonal matrix.

e) Scalar Matrix - A diagonal matrix is said to be a scalar matrix if its diagonal elements are equal.

For example - $\begin{bmatrix} \sqrt{3} & 0 & 0 \\ 0 & \sqrt{3} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}$ is a Scalar Matrix.

f) Identity Matrix - A square matrix in which the elements in the diagonal are 1 and rest are zeroes.

For example - $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

VIBA CLASSES

FORMULAS OF CONTINUITY AND DIFFERENTIABILITY

1. $\lim_{x \to a} \frac{x^n - a^n}{x - a} = nx^{n-1}$

2. $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

3. $\lim_{\theta \to 0} \cos\theta = 1$

4. $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

5. $\lim_{x \to 0} \frac{\log(1+x)}{x} = 1$

6. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

7. When limit exists $f(a) = LHL = RHL$

8. $y = x^n \quad dy/dx = nx^{n-1}$

9. $y = e^x \quad dy/dx = e^x$

10. $y = a^x \quad dy/dx = a^x \log a$

11. $y = x \quad dy/dx = 1$

12. $y = \sqrt{x} \quad dy/dx = 1/2\sqrt{x}$

13. $y = \frac{1}{x} \quad dy/dx = \frac{-1}{x^2}$

14. $y = \frac{1}{x^2} \quad dy/dx = -2/x^3$

15. $y = \frac{1}{x^3} \quad dy/dx = \frac{-3}{x^4}$

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16. $y = \sin x$ $\quad dy/dx = +\cos x$

17. $y = \cos\theta$ $\quad dy/dx = -\sin x$

18. $y = \tan x$ $\quad dy/dx = \sec^2 x$

Product Rule = $\frac{d}{dx}(I)(II) + \frac{d}{dx}(II)(I)$

Quotient Rule = $\dfrac{\frac{d}{dx}(I)(II) - \frac{d}{dx}(II)(I)}{(II)^2}$

19. $y = \sec x$ $\quad dy/dx = \sec x \tan x$

20. $y = \operatorname{cosec} x$ $\quad dy/dx = -\operatorname{cosec} x \cot x$

21. $y = \cot x$ $\quad dy/dx = -\operatorname{cosec}^2 x$

22. $y = \sin^{-1} x$ $\quad dy/dx = \frac{1}{\sqrt{1-x^2}}$

23. $y = \cos^{-1} x$ $\quad dy/dx = \frac{-1}{\sqrt{1-x^2}}$

24. $y = \tan^{-1} x$ $\quad dy/dx = \frac{1}{1+x^2}$

25. $y = \cot^{-1} x$ $\quad \frac{dy}{dx} = \frac{-1}{1+x^2}$

26. $y = \sec^{-1} x$ $\quad dy/dx = \frac{1}{x\sqrt{x^2-1}}$

27. $y = \operatorname{cosec}^{-1} x$ $\quad dy/dx = \frac{-1}{x\sqrt{x^2-1}}$

28. $y = \log x$ $\quad dy/dx = \frac{1}{x}$

29. $\log(a \times b \times c) = \log a + \log b + \log c$

30. $\log\left(\frac{a}{b}\right) = \log a - \log b$

31. $\log a^b = b \log a$

32. $\log_e e = 1$

33. $\log 1 = 0$

34. $\log 10 = 1$

VIBA CLASSES

INTEGRATION

Category 1

1.) $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

2.) $\int e^x dx = e^x + c$

3.) $\int a^x dx = \frac{a^x}{\log a} + c$

4.) $\int \sin x \, dx = -\cos x + c$

5.) $\int \cos x \, dx = \sin x + c$

6.) $\int \sec x \tan x \, dx = \sec x + c$

7.) $\int \operatorname{cosec} x \cot x \, dx = -\operatorname{cosec} x + c$

8.) $\int \frac{1}{x} dx = \log|x| + c$

9.) $\int \sec^2 x \, dx = \tan x + c$

10.) $\int \operatorname{cosec}^2 x \, dx = -\cot x + c$

Category 2 - Tricks

1.) $\int \sin^2 x \, dx = \int \frac{1-\cos 2x}{2} dx$

2.) $\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} dx$

3.) $\int \tan^2 x \, dx = \int (\sec^2 x - 1) \, dx$

4.) $\int \cot^2 x \, dx = \int (\operatorname{cosec}^2 x - 1) \, dx$

Category 3 -

1.) $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \log\left|\frac{x-a}{x+a}\right| + c$

2.) $\int \frac{1}{a^2-x^2} dx = \frac{1}{2a} \log\left|\frac{a+x}{a-x}\right| + c$

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3.) $\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

4.) $\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \log\left|x + \sqrt{x^2-a^2}\right| + c$

5.) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

6.) $\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \log\left|x + \sqrt{x^2+a^2}\right| + c$

Category 4 Useful for Ch - 8 and 10

1.) $\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2-a^2}\right| + c$

2.) $\int \sqrt{x^2+a^2}\,dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2+a^2}\right| + c$

3.) $\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$

Category 5 By Parts

(Product Rule of Integration)

1.) $\int \underset{\text{I}}{f(x)}\,\underset{\text{II}}{g(x)}\,dx = \text{I}\int \text{II}\,dx - \int\left(\text{I}'\int \text{II}\,dx\right)dx$

2.) $\int e^x\left[f(x) + f'(x)\right]dx = e^x f(x) + c$

Category 6 Partial fraction

1.) $\int \frac{px+q}{(x-a)(x-b)}\,dx = \frac{A}{x-a} + \frac{B}{x-b}$

2.) $\int \frac{px+q}{(x-a)(x-b)(x-c)}\,dx = \frac{A}{x-a} + \frac{B}{x-b} + \frac{C}{x-c}$

3.) $\int \frac{px+q}{(x-a)^2(x-b)}\,dx = \frac{A}{x-a} + \frac{B}{(x-a)^2} + \frac{C}{x-b}$

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4.) $\int \frac{px+q}{(x^2+bx+c)(x-d)} dx = \frac{Ax+B}{x^2+bx+c} + \frac{C}{x-d}$

5.) $\int \frac{px+q}{(x^2+bx+c)(x-d)^2(x+c)} dx = \frac{Ax+B}{x^2+bx+c} + \frac{C}{x-d} + \frac{D}{(x-d)^2} + \frac{E}{x+c}$

Category 7

1.) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1}x + c$
OR
$-\cos^{-1}x + c$

2.) $\int \frac{1}{1+x^2} dx = \tan^{-1}x + c$ OR $-\cot^{-1}x + c$

3.) $\int \frac{1}{x\sqrt{x^2-1}} dx = \sec^{-1}x + c$ OR $-\operatorname{cosec}^{-1}x + c$

Category 8

1.) $\int \tan x \, dx = -\log(\cos x)$ or $\log|\sec x|$

2.) $\int \cot x \, dx = \log(\sin x)$ or $-\log(\operatorname{cosec} x)$

3.) $\int \sec x \, dx = \log|\sec x + \tan x|$

4.) $\int \operatorname{cosec} x \, dx = \log|\operatorname{cosec} x - \cot x|$

VIBA CLASSES

VECTOR

1) Standard form = $x\hat{i} + y\hat{j} + z\hat{k} = \vec{a}$

2) Magnitude $= |\vec{a}| = \sqrt{x^2+y^2+z^2}$

3) Unit vector $= \hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$

4) When two points are given

$A(x_1, y_1, z_1) \qquad B(x_2, y_2, z_2)$

$$\vec{a} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{i} + (z_2 - z_1)\hat{k}$$

5) Section formulas m:n

Internally $\vec{R} = \dfrac{m\vec{b} + n\vec{a}}{m+n}$

Externally $\vec{R} = \dfrac{m\vec{b} - n\vec{a}}{m-n}$

6) Projection vector of $\vec{a}$ on $\vec{b}$ = $\vec{a} \cdot \dfrac{\vec{b}}{|\vec{b}|}$

7) Direction Ratio's

x, y, z = Direction Ratio

$\vec{a} = x\hat{i} + y\hat{i} + z\hat{k}$

$$\hat{a} = \frac{x\hat{i} + y\hat{i} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$$

Direction Cosines

$l = \cos\theta$, $m = \cos\alpha$, $n = \cos\beta$

$$l = \frac{x}{\sqrt{x^2+y^2+z^2}} \qquad n = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$m = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

8) Area of $\Delta = \frac{1}{2}|\vec{a} \times \vec{b}|$

9) Area of llgm $= |\vec{a} \times \vec{b}|$

10) $l^2 + m^2 + n^2 = 1$

11) Dot Product

- $\vec{a} \cdot \vec{b}$ = Scalar
- $\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$
- If $\vec{a} \perp \vec{b}$
  $\vec{a} \cdot \vec{b} = 0$
- $\vec{a} \cdot \vec{a} = |\vec{a}|^2$

Cross Product

- $\vec{a} \times \vec{b}$ = Vector
- $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|}$
- If $\vec{a} \parallel \vec{b}$
  $\vec{a} \times \vec{b} = 0$
- $\vec{a} \times \vec{a} = 0$

VIBA CLASSES

3D GEOMETRY

1. Direction Ratios

a, b, c

Direction Cosines

l, m, n

$$l^2 + m^2 + n^2 = 1$$

Direction Cosines

- Angle given

$l = \cos\theta$

$m = \cos\beta$

$n = \cos\gamma$

- (from direction ratios)

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}}$$

$$m = \frac{b}{\sqrt{a^2+b^2+c^2}}$$

$$n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

- Two Points given

$$l = \frac{x_2 - x_1}{PQ}$$

$$m = \frac{y_2 - y_1}{PQ}$$

$$n = \frac{z_2 - z_1}{PQ}$$

2. Equation of Line

Passing through the Point + Direction Ratio given

V.F. $\vec{r} = \vec{a} + \lambda\vec{b}$

C.F. $\frac{x - x_1}{a} = \frac{y - y_1}{b} = \frac{z - z_1}{c}$

Passing through two points

V.F. $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

C.F.

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

3. Angle between two lines

C.F. $\cos\theta = \left| \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\,\sqrt{a_2^2+b_2^2+c_2^2}} \right|$

If lines are $\perp$

$a_1a_2 + b_1b_2 + c_1c_2 = 0$

If lines are $\parallel$

$\dfrac{a_1}{a_2} = \dfrac{b_1}{b_2} = \dfrac{c_1}{c_2}$

V.F. $\cos\theta = \left| \dfrac{\vec{b_1}\,\vec{b_2}}{|\vec{b_1}|\,|\vec{b_2}|} \right|$

4. Distance between lines

Two skew lines

$$d = \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|b_1 \times b_2|} \right|$$

$$d = \frac{\begin{vmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}}{\sqrt{(b_1c_2 - b_2c_1)^2 + (c_1a_2 - c_2a_1)^2 + (a_1b_2 - b_1a_2)^2}}$$

Two $\parallel$ lines

$$d = \left| \frac{\vec{b} \times (\vec{a_2} - \vec{a_1})}{|\vec{b}|} \right|$$

5. Equation of Planes

(A) Normal given
Distance from the origin given

V.F. $\vec{r}\cdot\hat{n} = d$

C.F. $lx + my + nz = d$

(B) Normal vector + Passing through Pt given

V.F. $(\vec{r}-\vec{a})\cdot\vec{N} = 0$

C.F. A

$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$

(C) 3 Pts Non-Collinear

V.F. $(\vec{r}-\vec{a})\cdot[(\vec{b}-\vec{a})\times(\vec{c}-\vec{a})] = 0$

C.F.
$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ x_3-x_1 & y_3-y_1 & z_3-z_1 \end{vmatrix} = 0$$

Intercept form

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

6. Equation of Plane through Intersection of two given planes

V.F. $\vec{r}\cdot(\vec{n_1} + \lambda\vec{n_2}) = d_1 + \lambda d_2$

C.F. $(A_1x + B_1y + C_1z - d_1) + \lambda(A_2x + B_2y + C_2z - d_2) = 0$

Coplanarity of two Lines

If
$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix} = 0$$

then two lines are on the same planes

7. Angle between two Planes

$$\cos\theta = \left| \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2+B_1^2+C_1^2}\,\sqrt{A_2^2+B_2^2+C_2^2}} \right|$$

$$\cos\theta = \left| \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|} \right|$$

If two planes are ⊥

$$A_1A_2 + B_1B_2 + C_1C_2 = 0$$

If two planes are ||

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$$

8. Distance from the point to the plane

$$d = \left| \frac{Ax_1 + By_1 + Cz_1 - D}{\sqrt{A^2+B^2+C^2}} \right|$$

9. Angle between lines and plane

$$\cos\theta = \left| \frac{\vec{b}\cdot\vec{n}}{|\vec{b}|\,|\vec{n}|} \right|$$

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